SOME PERSPECTIVES ON PENSION VALUATION AND FUNDING: FINANCIAL MANAGEMENT CONSIDERATIONS

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ABSTRACT. . . The purpose of this paper is to compare and contrast the methodologies for incorporating pension valuation and funding into financial management. A review of the literature from Accounting, Actuarial Science, and Finance, indicates differences in the valuation, funding and recognition of pension plans based on ideological perspectives. One issue which underlies all research in the pension benefits area is the nature of guarantees mandated under ERISA and provided by the Pension Benefit Guarantee Corporation (PBGC). This investigation examines the role PBGC may provide in clarifying how to measure and recognize pension plan costs and future liabilities. In addition, this paper analyzes the position workers maintain in their pension plans, as economic agents, and the impact pension valuation and funding may have on the size of the firm and the substitution of capital for labor. The examination concludes with a summary of how financial managers may deal with the controversy over pension plan valuation and funding.

Introduction

Pension plans are designed to meet the needs of two constituencies. Firms provide the retirement benefits, and employees readily accept them as a form of deferred compensation. Several factors motivate this exchange of benefits and costs. Companies offer pension plans to improve employee relations, meet union demands, and take advantage of existing tax regulations to reward key workers. A firm can alter the age distribution of its work force by providing retirement incentives, thereby increasing advancement opportunities for young workers, and rewarding those who stay many years with the company. Employees view pensions as a way of saving for retirement with tax deferred dollars. If the rules of exchange apply to pension planning, the two groups will negotiate to the point where their marginal net benefits are equal. To the extent that the firm generates higher marginal revenue above marginal cost, there may be justification for employees negotiating higher wages and deferred benefits.

However, if pension or wage benefits become too costly the firm may be obliged to alter production or methods of compensation.

In exchanging current labor for future retirement benefits, workers assume various risks associated with the pension plan. A defined contribution plan involves no fund default risk, however, employees bear the burden of inflation and loss in the real value of retirement benefits. Under a defined benefit plan, workers may acquire risk associated with the firm's commitment to fund the pension. These workers have little inflation risk on future retirement income. Employees and PBGC have a lien on the firm's net worth in the event of plan default. Further, companies that fund a major portion of their future pension liabilities early may lower the default risk for workers. Employees that view pension default as a possibility may negotiate to have a defined contribution, rather than a defined benefit plan. Conversions may also be stimulated when inflation expectations decline. Workers are free to bargain a better compensation contract, if the risks and their negotiating strength are sufficiently large to permit change.

The ability of firms and workers to contract retirement benefits may depend on the quality of information concerning pension costs, assets, and liabilities. In addition, PBGC, as an insurer of residual claims on future pension benefits, also depends on data concerning defined benefit plans. There are two sources of information on pension plans, the actuaries and the accountants. Accounting data are generated from the auditor's role of attesting to the financial position of the firm. Accountants are concerned with the valuation of the pension plan as it relates to the meeting of future retirement obligations. Actuarial data are developed from the actuary's position as a fund administrator. The actuary provides a method or set of techniques suitable to the firm for amortizing

the cost of retirement benefits over time. Thus, the actuary may be more interested in funding, rather than valuation considerations in evaluating pension plans. Given the difference in perspectives, it is not unlikely that accounting and actuarial data vary. Unfortunately, discrepancies in the data may obscure the measurement of retirement benefits and costs for both the firm and workers.

Investors consider both pension funding and valuation in evaluating firm risk. If the firm is viewed on a going-concern basis, unfunded pension liabilities may represent a future financial burden to the company. The actuarial cost method chosen by the actuary for amortizing pension costs may indicate how substantial future pension liabilities are. Figures 1 and 2 illustrate how reported pension liabilites may change when switching from one pension cost method to another. Given a $15,000 deficiency to be amortized over the next five years, Figure 1 shows what the firm may contribute under the entry age normal and accrued benefit cost methods. Within five years, the liability is paid off completely under both cost techniques. However, entry age normal produces a more rapid reduction of the initial $15,000 liability. This difference in the rate of payoff generates changes in reported pension liabilities as the firm moves from one cost method to another during amortization.

Figure 2 shows the effect of switching between entry age normal and accrued benefit cost methods at the end of the second year. In Case 1, the firm starts with entry age normal cost and then changes to the accrued benefit cost method. The firm declares a $3,000 surplus at the end of the second year. The first two years under entry age normal costing have reduced the pension liability $3,000 more than what is required under the accrued benefit formulation.

Figure 1.

Reported Pension Liabilities Under the Entry Age Normal and Accrued Benefit Cost Methods

Entry Age Normal Cost (assuming a $15,000 initial pension liability):

	0	1	2	3	4	5 years
Employer's Contribution		$ 3,000	$3,000	$3,000	$3,000	$3,000
Pension Liability	$15,000	$12,000	$ 9,000	$ 6,000	$ 3,000	0

Accrued Benefit Cost (assuming a $15,000 initial pension liability):

	0	1	2	3	4	5 years
Employer's Contribution		$ 1,000	$ 2,000	$ 3,000	$ 4,000	$ 5,000
Pension Liability	$15,000	$14,000	$12,000	$ 9,000	$ 5,000	0

Source: A. Frank Thompson, Yong H. Kim, and Philip W. Glasgo, "Pension Liability Reporting Under ERISA," The International Employee Benefits Journal, Vol. 8, #1(March 1983), p. 5.

Figure 2.

Changes in Reported Pension Liabilities When Switching Pension Cost Methods

Case 1. At the end of the second year, the firm switches from using entry age normal to the accrued benefit cost method.

	0	1	2	3	4	5
						years
Employer's Contribution		$ 3,000	$ 3,000	$ 3,000	$ 4,000	$ 5,000
			†			
Pension Liability	$15,000	$12,000	$12,000	$ 9,000	$ 5,000	0
			†			
Pension Surplus			$ 3,000			

Case 2. At the end of the second year, the firm switches from using accrued benefit cost to the entry age normal method.

	0	1	2	3	4	5
						years
Employer's Contribution		$ 1,000	$ 2,000	$ 3,000	$ 3,000	$ 3,000
			†			
Pension Liability	$15,000	$14,000	$ 9,000	$ 6,000	$ 3,000	0
			†			
Additional Pension Liability			$ 3,000			

Source: A. Frank Thompson, Yong H. Kim, and Philip W. Glasgo, "Pension Liability Reporting Under ERISA," The International Employee Benefits Journal, Vol. 8, #1(March 1983), p. 5.

Case 2 demonstrates what happens when the firm begins amortization with accrued benefit cost and then moves to entry age normal cost. The accrued benefit cost method defers payment of a large portion of the pension liability to years three, four, and five. Consequently, very little of the liability is paid in the first two years of amortization under accrued benefit cost. When the firm switches from accrued benefit cost to entry age normal, there is an additional pension liability of $3,000. This liability arises from the fact that entry age normal is a much more conservative amortization scheme and requires an early reduction of pension liabilities.

Thus, very different surpluses or liabilities can be reported as a result of different methods of amortization or changes from one method to another. Accordingly, contribution levels can be altered almost at will, the reported liability may bear little resemblance to the actual obligation and comparison of liabilities from firm to firm can be illusory.

The Pension Benefit Guarantee Corporation (PBGC) is in the business of insuring defined benefit pension plans against default. PBGC assumes a portion of the risks associated with pension plan failure, in exchange for annual fees charged to sponsoring companies. ERISA states that PBGC can place legal claim on a company's assets up to 30% of net worth, whenever a defined benefit plan is terminated and the firm's pension fund assets are inadequate to provide retirement benefits. The PBGC's claim on corporate assets have the status of a tax lien, which places the interests of other creditors and stockholders subordinate to PBGC. Thus, a firm's shareholders assume a financial liability for retirement benefits to employees up to 30% of their net worth position in the company.

A major issue in the evaluation of pension risk is the nature of the insurance PBGC provides. Insurance is designed to transfer risk from an individual entity (e.g., a firm's pension plan) to a collective organization (e.g., PBGC), based on an equitable sharing of losses. Underwriting separates insureds into distinct risk categories, and premiums are calculated to directly reflect risk. In order to establish underwriting standards, an insurer develops a standard contract to cover a set of risks to be insured. Reserves are based on an actuarial determination of premium income, expected losses, and expenses over the insuring period. PBGC as an agency created by Congress, must accept all prospective pension plan risks regardless of their insurance characteristics. Firms may select from any one of five different actuarial cost methods to report the financial position of their pension. These companies may change their actuarial cost methods from year to year. Given the variations in pension surpluses and deficits that occur from using alternative cost techniques, PBGC may have difficulty evaluating the size and nature of the risks it accepts. If PBGC is unable to properly underwrite and rate pension plan risk, the workers and shareholders are less likely to understand the nature of the guarantees PBGC offers. Workers may hold a contingent risk in the financial solvency of PBGC. Shareholders may have to assess the liklihood of future premium increases on PBGC insurance to pay the cost of improperly underwriting the poorer pension risks. The flat premium assessment per worker for PBGC insurance leaves no room for classifying and charging firms equitably on the basis of pension plan risk. Unfortunately, the development of a risk-related premium for PBGC insurance would increase pension expenses on firms already strapped for cash. Currently, firms with healthy pension plans may be subsidizing companies with high pension default risk through the premium structure of PBGC insurance.

Review of Current Literature on Pension Funding and Valuation

A review of the literature from Accounting, Actuarial Science, and Finance, indicates differences in the valuation, funding and recognition of pension plans based on ideological perspectives. Actuarial research, sponsored by the Pension Research Council, (Marples [11], Trowbridge and Farr [20]) focuses on the funding aspects of pensions. These books develop the theory behind pension funding based on several alternative actuarial models. These models incorporate such factors as rates of withdrawal, disability, salary progression, interest, and expenses in the funding of pensions. Trowbridge and Farr [20] explore the impact population stationarity and inflation have on pension funding. Models of mature and immature funds are presented in both inflationary and noninflationary economies. Trowbridge and Farr [20] demonstrate how pension fund conditions change as a result of the aging of an employer's work force and the presence of inflation. Both factors tend to increase the financial burden on the firm in funding future retirement benefits. Actuarial literature emphasizes how firms may adjust current and future employer contributions to meet changing conditions within their pension plans.

Accounting research (Coopers and Lybrand [4], Lucas and Hollowell [10]) focuses on the measurement or valuation of pension liabilities. Valuation is an important topic to accountants because of their responsibility for accurately reporting financial position. Accountants have two views concerning the recognition of pension liabilities. First, pension benefits are a form of deferred compensation which obligate the firm to the employee and therefore, represent a liability that should be included on the balance sheet. This position is currently held by FASB in their Preliminary Views. An alternative outlook, Coopers and Lybrand [11], is that retirement benefits are funded by the firm

through the pension plan. The obligation for providing pension benefits rests with the plan, not the firm. Accordingly, the valuation for future pension benefits should be noted as an off balance sheet item. The latter view is consistent with APB No. 8, and has the support of several Big 8 accounting firms.

Lucas and Hollowell [10] provide the argument for recognizing pension liabilities on the balance sheet. Support for this position comes from recent cases involving Facet Enterprises, a subsidiary of The Bendix Corporation, and Kaiser Steel. In 1976, The Bendix Corporation wanted to spin-off, Facet Enterprises, and have the subsidiary terminate its underfunded pension plan. Bendix planned to relieve itself of any pension fund obligation to Facet workers by having the separated firm apply to PBGC for claims settlement. The Bendix Corporation could not legally absolve itself from the pension obligation to workers in the Facet subsidiary. Later, in 1980, Kaiser Steel was contemplating liquidation at $44 a share. However, when the unfunded vested pension liability and guaranteed health benefits were added into the valuation, the company discovered its firm value was well below $44 per share. The presence of a substantial retirement liability dampened Kaiser Steel's interest in liquidation. Lucas and Hollowell [10] use these cases to demonstrate how pension liabilities represent a real financial obligation for the firm. The authors argue that: (1) the fact that contributions are scheduled for payment in the future does not mean the obligation should go unrecorded, many reported liabilities have scheduled future payments, (2) pension obligations have been

held to be legally enforceable, (3) since pension plan amendment is an event that changes the firm's obligation, financial statements should reflect those changes according to Financial Accounting Concepts Statement No. 1.

Finance studies of pension plans may be divided into two separate lines of research. A great number of investigations (Arnott and Gersovitz [1], Bulow [3], Feldstein [6], Feldstein and Seligman [7], Langeteig, Findlay, and DaMotta [9], Miller and Scholes [13], Tepper and Affleck [15], Treynor [18]) analyze how pension liabilities may be incorporated into the corporate finance framework of the firm. These studies consider how pension obligations and funding requirements impact on meeting the company's overall objective of maximizing shareholder wealth. A second set of papers (McKenna and Kim [12], Tepper [14]) deal with the management of pension fund assets in meeting the firm's retirement goals for workers. Such investigations examine how the firm's investment strategies may be coordinated with the development of the pension plan.

Investigations on the inclusion of pensions into corporate financial structure analyze the significance pension liabilities have on firm valuation. The evaluation of pension obligations and the nature of PBGC insurance are important variables in this assessment. Arnott and Gersovitz [1] provide a dynamic general equilibrium model of the pension contract, which suggests that corporate structure and the employment contract are interdependent. Their research considers retirement liabilities as true obligations for the firm which alter financial risk. Bulow [3] further defines the contractual obligation the firm has for generating retirement benefits to workers in light of PBGC

insurance. Given the existence of government insurance, Bulow [3] asserts that the firm's retirement obligation is the lesser of pension assets minus pension liabilities, or thirty percent of net worth.

Miller and Scholes [13], Treynor [18], and Langetieg, Findlay and DaMotta [9] examine what effect PBGC insurance has on limiting the firm's pension plan risk. These papers utilize option pricing theory to demonstrate that pension obligations may be divided into two elements: (1) the pension claim, which represents a liability for the firm, and (2) a "pension put" held by the employer on the underlying assets of the pension claim, with strike price equal to the value of the claim. In the event of a plan default, the firm can put the pension claim to PBGC. The literature (Treynor [18] and Langetieg, Findlay and DaMotta [9]) argues for the pension claim resting with the employer, but the ultimate risk of plan default being borne by PBGC. Thus, the firm's maximum potential loss in the event of plan default or termination is thirty percent of net worth. The relationship between the firm and PBGC is similar to that of ceding and reinsurance parties on a stop-loss treaty. Losses for the ceding insurer (i.e., firm) cease when the claims exceed the level specified (i.e., thirty percent of net worth) where the reinsurer accepts the risk.

Feldstein [6] and Feldstein and Seligman [7] present empirical results on the influence unfunded pension liabilities have on stock prices. These studies relate market value of the firm's equity, earnings growth and value of the firm's debt to reported unfunded vested benefits for approximately 200 manufacturing concerns. Their findings indicate that share prices fully reflect the value of reported unfunded vested obligations. The authors conclude that the market may be efficient in reacting to a firm's reported pension liabilities. However, the investigations leave open the question of what impact changes in actuarial cost methods and reporting of future pension benefits have on share prices.

Papers by McKenna and Kim [12] and Tepper [14] develop management models which may be helpful in setting investment policy for pensions. Tepper [14] integrates corporate tax considerations into the firm's decision to fund and invest pension assets. The major conclusion from this study is that companies should fully fund the pension and invest all plan assets in corporate bonds. This result is based on the fact that investment return on bond holdings embedded in a pension are nontaxable, while interest on corporate debt is a tax deductible expense. The firm borrows through the capital market to fully fund its pension, and these monies are arbitraged using the spread between the firm's tax adjusted loan rate and the return on corporate bonds in the pension plan. Tepper [14] claims the shareholders may be better off because the firm's value increases through arbitraging pension assets. In addition, investors may offset any adverse consequences from the bond mix in the pension by purchasing equity for their own portfolio. The implication is that the shareholder's interest in the company may be separated from the management of the pension. Tepper [14] assumes that stockholders are capable of understanding issues related to the valuation, funding mix and investment policy of the pensions in the securities they hold.

McKenna and Kim[12] use simulation methods to create investment policies for company pension plans that are consistent with the risk preferences of the firm's management. The emphasis of this paper is on tying together the risk preferences of the firm with the cost and funding objectives of the pension plan. The study investigates the choice of a long-run investment policy for a salary-based defined benefit pension plan. The selection of a long-run time period

obviates the need to consider the short-run problem of whether to recognize future pension liabilities immediately. This paper's contribution to pension research appears to be in the coordination of managerial objectives and risk preferences for pension plan development with the actuarial assumptions and modeling of retirement benefits.

Pension research appears to follow closely the particular interests of the disciplines that interact with defined benefit pension plans. Finance literature emphasizes two issues: (1) how or whether pension liabilities influence the value of the firm and ultimately, share price and (2) what methods are available to the firm for managing and funding pension plans. Accountants are concerned with the valuation and reporting of pension expenses and liabilities. Actuarial research centers on how to adequately fund defined benefit plans taking into consideration such microeconomic factors as rates of withdrawal, disability, increases in salary, and differences in benefit definitions (i.e., final average salary versus career average). A common issue which seems to transcend ideological perspective is the evaluation of the health of the pension plan. Can the pension through actuarial funding, accounting for costs and liabilities, and investing pension assets meet the retirement obligation the firm has to its workers? If a company can not fulfill its pension commitment, then to what extent can and will PBGC guarantee the retirement promises of the firm?

The development by PBGC of a standard indemnity contract for insuring pension plan default may eliminate some of the uncertainty surrounding the evalution of pensions. PBGC, as the insurer of last resort for defined benefit plans, could impose rules for determining pension liabilities and costs for insurance purposes. In addition, the agency could limit coverage to various level

amounts and permit each firm to choose the type of coverage desired. The insurance protection would be priced according to the size of coverage and amount of risk assumed by PBGC. A common method for measuring pension liabilities, along with graded insurance against pension default may permit investors, and researchers in Accounting, Finance, and Actuarial Science to guage the importance of a firm's pension obligation. Presently, only firm comparisons made on the basis of unfunded vested benefits are possible. A uniform PBGC insurance contract providing fully reserved benefits in the event of default would allow an accurate appraisal of the employee's, employer's and investor's position with respect to a firm's pension.

Determining PBGC's Retention Limit on Pension Insurance

Before the Pension Benefit Guarantee Corporation can determine the premiums and reserves to meet future insurance claims, it must first decide on how much coverage to retain. The insuring of pensions involves interest rate risk on securities, investment, and bad management risk on pension portfolios, all of which may depend on economic conditions. The dynamic and fundamental nature of these risks imply that PBGC may be unable to retain responsibility for catastrophic coverage. Currently, PBGC has a statutory line of credit with the Treasury which could be used in the event of massive withdrawals on the corporation. This agency's capacity to retain insurance will increase with the size of its reserves and the premium it charges member firms. A control model will be used to identify the relationship involved in determining PBGC's retention limit on insuring defined benefit pension plans.

A way of examining the effect that operating capacity has on retention of insurance is to consider a dynamic model of reinsurance for PBGC. In addition to quantifying the connection between retention limit and PBGC fund size, such a model may prove useful to finding a suitable credit line for excess coverage. PBGC's investable funds can be divided into two parts. One consisting of technical reserves R' to meet current claims experience. The other fund represents free reserves R'' used to meet unexpected adverse deviations away from expected risk.[1] R'' directly affects the amount of insurance PBGC can absorb into its risk portfolio. Let,

$K_1(t)$ = the amount in R' at time t,

$K_2(t)$ = the amount in R'' at time t.

These two funds will earn individual rates of return r_1 and r_2 over time. Thus,

$r_1(t)$ = the short-run interest rate on money held in K_1,

$r_2(t)$ = the long-run interest rate on money held in K_2.

In addition to these returns, the PBGC will experience increases (or decreases) in equity as a result of trades on long-term assets held in $K_2(t)$. Let,

$r_3(t)$ = the increase (or decrease) in the value of long-term assets.

If F(X(t)) represents the cumulative distribution function for claims, then the pure risk premium is defined to be:

$$d(t) = \int_0^\infty x(t)\, F'(x(t)),$$

where $x(t) = K_1(t) + K_2(t) + P(t)$,

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[1] Let F(x) = the cumulative distribution function, which represents the probability the amount of claims paid under a portfolio of insurance contracts will not exceed the random variable x. $R' = \int_0^\infty xF'(x)\,dx$ and R'' = a safety reserve, an amount held in addition to R' to meet contingent losses greater than expected claims.

$P(t)$ = premiums collected in underwriting period t, and

$d(t)$ = demand for cash to meet claims in period t, ($d(t) \in [0,\infty)$).

The controlling variable is $u(t)$, the amount of insurance coverage ceded to the credit line at time t. $|u(t)| < M$, where M is an upper bound on the amount of credit that can be drawn upon in any period t. PBGC incurrs a penalty expense α for each credit line transaction. The set of state equations are as follows:

$$\dot{K}_1(t) = r_1(t)\cdot K_1(t) - d(t) + u(t) - \alpha|u(t)| + r_2(t)\cdot K_2(t). \quad (1)$$

$$\dot{K}_2(t) = r_3(t)\cdot K_2(t) - u(t). \quad (2)$$

According to equation (1) a change in technical reserves in any time t results from:

(i) an interest return $r_1(t)\cdot K_1(t)$ on short-term holdings,

(ii) a decrement $d(t)$ based on the payment of claims in time t,

(iii) an increase (or decrease) by using the credit line,

(iv) the payment of transactions costs $\alpha|u(t)|$, and

(v) a return on long-term assets, $r_2(t)\cdot K_2(t)$.

Differential equation (2) represents changes to the long-term fund from:

(i) an increase (or decrease) in the value of assets over time, and

(ii) an increase (or decrease) caused by use of the credit line.

PBGC's objective is to maximize the functional $[K_1(t) + K_2(t)]$ with respect to the controller $u(t)$ over time subject to equations (1) and (2). The canonical representation of this system is given as:

$$\max_{u(t)} \{[1,1]\cdot \vec{K}(t)\}$$

Subject to:

$$\dot{\vec{K}}(t) = A\vec{K}(t) + bu(t) + c \qquad (3)$$

where

$$K(t) = \begin{bmatrix} K_1(t) \\ K_2(t) \end{bmatrix},$$

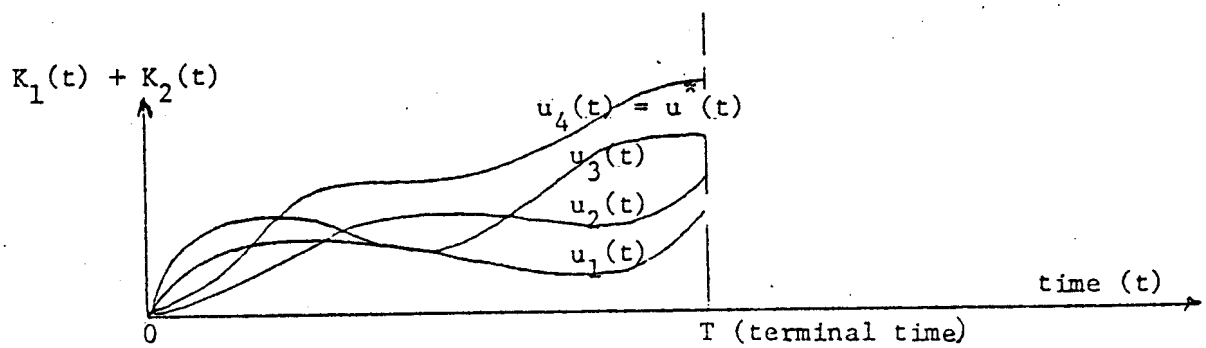
$$A = \begin{bmatrix} r_1(t) & r_2(t) & 0 \\ 0 & 0 & r_3(t) \end{bmatrix},$$

$$b = \begin{bmatrix} 1 - \alpha \\ -1 \end{bmatrix}, \quad \text{and} \quad c = \begin{bmatrix} -d(t) \\ 0 \end{bmatrix},$$

with claims experience d(t) exogeneously defined. The insurer wants to find the optimal decision rule $u^*(t)$ from a set of rules $\{u_i(t), \text{ for all } i\}$, which leades to a maximum value for $[K_1(t) + K_2(t)]$ at terminal time T. Figure 1 portrays this selection process graphically.

Figure 1

Selection of an Optimal Retention Plan



Selecting any $u_i(t)$ leads to some terminal value $[K_1(T) + K_2(T)]_i$. The set of admissible controls $u_i(t)$ is bounded by M, and the state system (equation (3)) is linear in u(t) and K(t) which guarantees a unique solution. The Hamiltonian can be defined as:

$$H[K_1(t),K_2(t),p_1(t),p_2(t),u(t),t]$$

$$= p_1(t)\ \{r_1(t)\cdot K_1(t) - d(t) + u(t) - \alpha|u(t)| + r_2(t)\cdot K_2(t)\} \tag{4}$$

$$+ p_2(t)\ \{r_3(t)\cdot K_2(t) - u(t)\}\ .$$

Applying Pontryagin's maximum principle, an optimal policy can be obtained from the system of equations defined by:

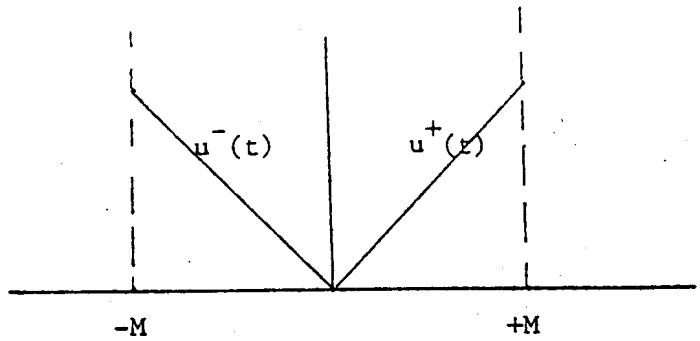
$$\frac{\partial H}{\partial u}[K_1(t),K_2(t),p_1(t),p_2(t),u(t),t] = 0.$$

The controller u(t) can take on both positive and negative values depending on whether the insurer is using or restoring the credit line in time period t. Since $|u(t)|$ is a discontinuous function, it is not possible to directly differentiate H. However, by suitably defining u(t) a derivative can be obtained. Let

$$u(t) = \begin{cases} u^+(t) - u^-(t) & \\ u^+(t)\cdot u^-(t) = 0 & \text{for all } u^+(t)\geq 0,\ u^-(t)\geq 0 \end{cases}$$

Figure 2 shows what this function looks like graphically.

The Decomposed Controller: u(t)



Withdrawals on the Credit Line

$u(t) = u^{+}(t)$, whenever the insurer is using the credit line,

$u(t) = u^{-}(t)$, whenever the insurer is restoring the credit line,

$u(t) = 0$, whenever the insurer neither uses nor restores the line of credit.

The matter of $|u(t)|$ is handled by making this expression equal to the sum of $u^{+}(t)$ and $u^{-}(t)$, i.e., $|u(t)| = u^{+}(t) + u^{-}(t)$. Under this new formulation the Hamiltonian is redefined to be:

$$H[K_1(t),K_2(t),p_1(t),p_2(t),u^{+}(t),u^{-}(t),t] \tag{5}$$

$$= p_1(t)\ \{r_1(t)\cdot K_1(t) - d(t) + u^{+}(t) - u^{-}(t) - \alpha(u^{+}(t) - u^{-}(t)) + r_2(t)\cdot K_2(t)\}$$

$$+ p_2(t)\ \{r_3(t)\cdot K_2(t) - (u^{+}(t) - u^{-}(t))\}.$$

Maximizing H with respect to $u^{+}(t)$ and $u^{-}(t)$:

$$\frac{\partial H}{\partial u^{+}(t)} = (1 - \alpha)p_1(t) - p_2(t), \tag{6}$$

$$\frac{\partial H}{\partial u^{-}(t)} = -(1 + \alpha)p_1(t) + p_2(t). \tag{7}$$

Since H is linear in $u^+(t)$ and $u^-(t)$ the solution defines an on-off switching policy where:

$$u^+(t) = \begin{cases} M \text{ if } \frac{\partial H}{\partial u^+(t)} > 0, \ (1-\alpha)p_1(t) - p_2(t) > 0 \\ 0 \text{ if } (1-\alpha)p_1(t) - p_2(t) \leq 0. \end{cases}$$

$$u^-(t) = \begin{cases} -M \text{ if } \frac{\partial H}{\partial u^-(t)} > 0, \ -(1+\alpha)p_1(t) + p_2(t) > 0, \\ 0 \text{ if } -(1+\alpha)p_1(t) + p_2(t) \leq 0. \end{cases}$$

The insurer (PBGC) acquires credit whenever $(1-\alpha)p_1(t) > p_2(t)$ and attains more insurance capacity if $p_2(t) > (1-\alpha)p_1(t)$. The values of the co-state or shadow variables $p_1(t)$ and $p_2(t)$ may be found by examining the system of adjoint equations defined to be:

$$\dot{p}_1(t) = -\frac{\partial H}{\partial K_1(t)} \tag{8}$$

$$\dot{p}_2(t) = -\frac{\partial H}{\partial K_2(t)} \tag{9}$$

$$\dot{p}_1(t) = -[r_1(t)\ p_1(t)\ ] \tag{10}$$

$$\dot{p}_2(t) = -[r_2(t)\ p_1(t) + r_3(t)\cdot p_2(t)] \tag{11}$$

Solving (10) for $p_1(t)$: $\int_t^T \frac{\dot{p}_1(t)}{p_1(t)}\, dt = -\int_t^T r_1(t)\, dt,$

which implies, $p_1(t) = \exp\{ \int_t^T r_1(t)\, dt \}$, (12)

since $p_1(t) = 1$.

Substituting equation (12) into equation (11):

$$\dot{p}_2(t) = -r_2(t) \exp\{ \int_t^T r_1(t)\, dt \} - r_3(t)\, p_2(t). \quad (13)$$

Using the fact that $p_2(t) = 1$ and the variation of constants formula:[2]

$$\dot{p}_2(t) = \{\exp(\int_t^T r_3(t)\, dt\}\{-\int_t^T r_2(t)\, [\exp\{\int_t^T r_1(t)\, dt + \int_t^T r_3(t)\, dt\}]dt\}. \quad (14)$$

This model describes a risk retention policy for PBGC consistent with the goal of optimizing insurance capacity over time. Given that PBGC sets a retention limit M and anticipates experiencing claims of d(t), then a time optimal policy for using a Congressional credit line on an excess-loss basis is defined by equations (6), (7), (12), and (14). Since M and d(t) are exogeneous variables, PBGC can perform sensitivity analysis on the optimal solution by varying these two parameters to determine their effect on K(t). Testing of the solution in this manner will indicate how dramatically PBGC's retention limit (M) may change as a result of increases or decreases in claims experience (d(t)) over time. This analysis may be used to determine future retention limits consistent with PBGC's attitudes toward risk and return.

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[2] Similar results hold for time optimal control models for stock portfolios, see A. Bensoussan, E. Gerald Hurst, Jr., and B. Naslund, Management Applications of Modern Control Theory, (Amsterdam: North-Holland Publishing, 1974).

An Analysis of the Externalities Involved In Providing Pension Benefits

Pensions define a relationship between firms and workers, whereby external benefits may be exchanged. Employees offer work in return for deferred compensation at retirement. Workers will accept this relationship, if there is reason to believe retirement benefits will be paid. Firms will continue the arrangement until it becomes too costly to fund benefits. Benefit plans are a voluntary means for employers to pay workers an implicit wage in the form of retirement income. If the wage bill meets or exceeds the firm's budget constraint, then pensions may be altered or eliminated.

Employee pension benefits may depend on two factors. The type of benefit paid to workers based on company employment, and the number of employees who share in the pension fund. A worker's utility function may be described as:[3]

$$u^b = u^b(B_1, B_2, \ldots B_N, N) \qquad (15)$$
$$u^b = u^b(B_1, N) + \ldots + u^b(B_N, N)$$

where u^b represents a twice differentiable, separable utility function,
B_i equals the amount of deferred benefits accorded to worker i,
N denotes the number of memebers in the pension.

The firm's cost function may be denoted as:

$$F = F[(B_1, N), (B_2, N), \ldots (B_N, N)] \qquad (16)$$
$$= F(B_1, N) + \ldots F(B_N, N)$$

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[3]James M. Buchanan, and William Craig Stubblebine, "Externality," Economica, (November 1962), pp. 371-372.

where F is a twice differentiable, separable cost function. Given these functional relationships, an analogy can be made between a pension and a club.[4] The firm's cost for providing pension benefits depends on the activities of each worker, B_i, and the size of the pension group, N. The addition of new members to the pension will affect the cost of providing benefits. The larger the membership in the pension the lower the cost of any single member, given the funding constraint on the firm. The size of the pension (club) determines the level of benefits that may be supplied to workers. Membership in the pension is an externality for both the employer and employees. The worker's benefits are ultimately determined by the claims on the pension fund.

Forming the Lagrangean and maximizing over all relevant variables, one can derive first order conditions for optimizing an individual worker's utility.

$$L = u^b(B_1,N) + \ldots + u^b(B_N,N) - \lambda(\bar{F} - [F(B_1,N) + \ldots + F(B_N,N)] \quad (17)$$

where $\bar{F}$ is the least upper bound on the level of pension expenditure.

$$\frac{\partial L}{\partial B_i} = \frac{\partial u^b}{\partial B_i}(B_i,N) - \lambda[\frac{\partial F}{\partial B_i}(B_i,N)] = 0. \quad (18)$$

for all i, and

$$\frac{\partial L}{\partial N} = \sum_{i=1}^{N} [\frac{\partial u^b}{\partial N}(B_i,N) - \lambda[\frac{\partial F}{\partial N}(B_i,N)] = 0. \quad (19)$$

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[4] The following development is based on the economic theory of clubs, see Buchanan [2].

The set of equations described in (18) implies that:

$$\frac{\frac{\partial u^b}{\partial B_i}}{\frac{\partial u^b}{\partial B_j}} = \frac{\frac{\partial F}{\partial B_i}}{\frac{\partial F}{\partial B_j}} \qquad (20)$$

or the marginal rates of substitution for benefits B_i and B_j must equal the marginal rates of substitution for the cost of those same benefits in exchange. Any marginal difference between B_i and B_j may be due to the higher wages or greater productivity of one worker over another.

Further, according to (18) and (19):

$$\frac{\frac{\partial u^b}{\partial N}}{\frac{\partial F}{\partial N}} = \frac{\frac{\partial u^b}{\partial B_i}}{\frac{\partial F}{\partial B_i}} \quad \text{for all } i \qquad (21)$$

which implies that:

$$\frac{\frac{\partial u^b}{\partial N}}{\frac{\partial u^b}{\partial B_i}} = \frac{\frac{\partial F}{\partial N}}{\frac{\partial F}{\partial B_i}} \qquad (22)$$

so that the marginal rate of substitution "in consumption" between the size of the pension sharing the benefit B_i, must equal the marginal rates of substitution for providing such a benefit.

The implication from relations (21) and (22) is that the firm will want to add new employees up to the point where the marginal benefits from employment just equal the marginal costs borne in funding retirement income. This condition maximizes each worker's utility function when facing a pension cost constraint.

One may conclude from (21) and (22) that firms with escalating pension costs may prefer to maintain or reduce existing employment levels to meet their pension obligations to workers. The results imply that there may be an optimal employment position for the firm based on the marginal equality of the costs and benefits to the pension from hiring an additional worker.

An examination of pension issues using production theory yields results similar to the preceding externality analysis. Consider a twice differentiable production function $Q = f(K,L)$ which describes the technical relationships between capital, K, and labor, L. Let r represent the return to capital. Divide the return to labor into two parts: (1) the direct wage return w_1 and (2) the implicit wage, w_2, for pension benefits. The cost function is given by:

$$C = rK + (w_1 + (1-T)w_2)L \tag{22}$$

where T = the firm's marginal tax rate.

The Lagrange function is:

$$V = f(K,L) - \lambda [\, rK + (w_1 + (1 - T)w_2)L]. \tag{23}$$

Maximizing L with respect to K and L yields the marginal rate of technical substitution.

$$\frac{\partial V}{\partial K} = \frac{\partial f}{\partial K} - \lambda r = 0 \tag{24}$$

$$\frac{\partial V}{\partial L} = \frac{\partial f}{\partial L} - \lambda[w_1 + (1 - T)w_2] = 0 \tag{25}$$

Equations (24) and (25) imply that:

$$\frac{\frac{\partial f}{\partial L}}{\frac{\partial f}{\partial K}} = \frac{w_1 + (1 - T)\, w_2}{r} \tag{26}$$

the marginal rate of substitution of capital for labor equals the ratio of their prices in production. The firm operates up to the level where the return to each factor of production equals its marginal product. If the return to labor increases relative to capital, the firm may substitute capital for labor in order to maintain optimal levels of production. Two other possibilities exist: (1) workers could agree to lower explicit wages, w_1, to offset increases in w_2 or (2) they might accept reduced pension benefits. Note also, that according to (26), government taxes may alter the firm's position on technical rates of substitution. The relation between the marginal rate of substitution and the firm's marginal tax rate is an inverse one. If the company is in a high tax bracket, the impact of expensing pension contributions lessens the firm's out of pocket costs for w_2.

One may conclude from this analysis that firms faced with higher than expected pension expenses, may attempt to become more capital intensive, substituting capital for labor. The reduction of wages w_1 or w_2 and the increases in output generated from higher capital may assure optimal production levels.

Conclusion

Concerns over the valuation and funding of corporate pensions provide the impetus for considerable research by accountants, actuaries, financial managers, and economists. This literature may be classified according to the disciplinary perspectives of the researchers. Accountants desire to set standards for reporting pensions on the balance sheet. The Preliminary Views published by the Financial Accounting Standards Board supports the notion

of having auditors use common cost techniques to recognize pension liabilities. Actuarial literature attempts to draw together various economic and demographic factors into a plan for funding pension benefits. Financial managers and economists try to measure the impact pensions may have on operating characteristics of the firm and the level of saving in the general economy.

A common theme throughout the literature is the issue of whether the retirement benefits to workers can be guaranteed. Congress, through the enactment of ERISA and the creation of PBGC speaks to that concern. However, experience since 1974 seems to indicate that the federal government can mandate neither the financial solvency of pensions nor the insurance capacity of a corporation such as PBGC to protect retirement benefits. A basic rule of underwriting is that the risk must be fully understood before it may be insured. Given five different actuarial cost methods, and the variations in wage and interest assumptions to report pension liabilities, it appears difficult to gauge the frequency and severity of the risks PBGC insures. In addition, the thirty percent loss limitation on the firm from pension default may encourage adverse selection. A firm may prefer termination of the pension with a known thirty percent loss of net worth, rather than an uncertain pension liability in the future.

This investigation indicates the need for common measures of pension liabilities and a standard risk-related excess loss insurance provided by PBGC. If the AICPA through FASB endorses a single method of reporting pensions on the balance sheet, such information would permit a fair assessment of each firm's pension plan. PBGC could use this data in setting rates for insuring pensions based on the risk characteristics of the plans. Insurance written on an excess loss basis with coinsurance provisions requiring firms to share pension losses, may avoid problems with adverse selection. Further, instead

of offering a high level of coverage for a flat premium, PBGC could provide graded protection with escalating premiums. The firm and workers could decide the extent of coverage to be purchased from PBGC.

The preceding analysis also shows what may happen under present pension funding and valuation procedures. Given the uncertainty over pension liabilities, firms and workers may seek alternatives to defined benefit plans. The development of defined contribution, employee stock ownership, and 401 (K) (2) benefit plans represent viable options in lieu of a defined benefit program. Employees may prefer higher wages, investing additional funds in an IRA, rather than participation in a firm's pension. Companies may desire retirement programs, such as defined contribution plans, that fix the firm's pension liability and eliminate the burden of supplying a set of benefits dependent on future rates of interest and wages. If corporations are unable to control pension costs and liabilities, they may take any one of the following actions: (1) terminate the plan to PBGC, (2) renegotiate to reduce pension benefits, or (3) try to decrease employment levels, substituting capital for labor to generate sufficient productivity to cover pension costs.

Pension plans exist for the benefit of the firm and its employees. These two groups negotiate over the level of retirement benefits the firm provides, and the increased productivity workers generate in return. The culmination of this process is a Pareto equilibrium position, whereby the employeees and the firm reach a pension agreement that can not be improved for one party without hurting the other's interests. If the pension contract no longer has value to either party, a renegotiation of benefits may take place or the contract terminated altogether. Currently, both companies and their workers may be

dissatisfied with defined benefit plans because of the uncertainty surrounding the valuation and funding of retirement benefits.

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